Date:

Chapter: Chapter 5:4 --> Complex Numbers

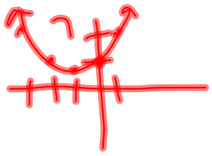
Objectives: Perform operations with pure imaginary numbers and complex numbers

Notes:

x	y
1	7
0	4
-1	3
-2	4
-3	7

$x = \frac{-b}{2a}$

$x = \frac{-2}{2(1)} \rightarrow -1$



Consider the graph of y = x^2 + 2x + 4. Notice how this graph has no x-intercepts and therefore does not have any roots. Does this mean there are no solutions?

$(-1)^2 + 2(-1) + 4$

*__Imaginary Unit__ (i) = Principal √-1 therefore i = √-1

*__Pure Imaginary Numbers__ = Square root of negative real numbers; for any positive real number b, $\sqrt{-b^2} = \sqrt{b^2}$ times √-1 = bi

---The Commutative and Associative Properties of Multiplication hold true for pure imaginary numbers---

*__Complex Numbers__ = An expression that contains both a real number and an imaginary number. (a ± bi) where a = real number and bi is the imaginary number

*__Complex Conjugates__ = Two complex numbers of the form a + bi and a - bi; the product of two conjugates is always a real number.

Square Root Property

--Method used to solve a quadratic equation--

If $x^2 = n$ then $x = \pm\sqrt{n}$ $x^2 = 121$ $x^2 = -121$

Powers of i

$i^2 \cdot i$ (-1) $i^2 \cdot i^2$ (-1)(-1)

$i^1 = i$	$i^2 = -1$	$i^3 = -i$	$i^4 = 1$
$i^5 = i$ ($i^2 \cdot i^2 \cdot i$)	$i^6 = -1$	$i^7 = -i$	$i^8 = 1$

Examples:

Ex. 1 - Simplify.

a) $\sqrt{-27}$

$\sqrt{-1}\cdot\sqrt{27}$

$i\cdot 3\sqrt{3}$

$3i\sqrt{3}$

b) $\sqrt{-216}$

$\sqrt{-1}\cdot\sqrt{216}$

$i\cdot 6\sqrt{6}$

$6i\sqrt{6}$

c) $\sqrt{-18}$

$3i\sqrt{2}$

d) $\sqrt{-125}$

e) $(2i^2)(3i^3)(-2i)$

$2(-1)(3)(-i)(-2)(i)$

$-12i^6$

$12(-i^2)$

$12(-1)(-1) = 12$

f) i^{63}

$(i^3)^{21} = -i$

g) $i^{21} = i$

$i^{20} = 1$

$i^{20}\cdot i^1 =$

$1\cdot i = i$

h) $(-2i)(-3i)(i)$

$6i^3 \rightarrow 6(-i)$

$-6i$

k) ~~3i 4i~~

$-i\cdot i\cdot(-1)\cdot i\cdot(-1)$

l) $\sqrt{-20}\cdot\sqrt{-12}$

$\sqrt{240}$

$4\sqrt{15}$

$\sqrt{-1}\cdot\sqrt{20}$

$2i\sqrt{5}\cdot\sqrt{-1}\cdot\sqrt{12}$

$2i\sqrt{5}\cdot 2i\sqrt{3}$

$4i^2\sqrt{15}$

$-4\sqrt{15}$

m) i^{31}

$i^{28}\cdot i^3$

$1\cdot(-i) = -i$

Ex. 2 - Add/simplify.

a) $(-2 + 5i) + (1 - 7i)$

$5x - 7x$

$-1 - 2i$

b) $(3 + 5i) + (2 - 4i)$

$5 + i$

c) $(4 - 2i) - (3 - 7i)$

$4 - 2i - 3 + 7i$

$1 + 5i$

Ex. 3 - Solve the quadratic equation.

a) $5y^2 + 20 = 0$

$5y^2 = -20$

$y^2 = -4$

$y = \sqrt{-4} \rightarrow \sqrt{-1}\cdot\sqrt{4}$

$i\cdot 2 = \pm 2i$

$x^2 = 121$

$x = 11, -11$

b) $4x^2 + 24 = 0$

$4x^2 = -24$

$x^2 = -6$

$x = \sqrt{-6}$ $\sqrt{-1}\cdot\sqrt{6}$

$x = \pm i\sqrt{6}$

c) $5x^2 = -150$

$x^2 = -30$

$x = \sqrt{-30} \rightarrow \sqrt{-1}\cdot\sqrt{30}$

$\pm i\sqrt{30}$

Ex. 4 - Find the values for x and y that make the equation true.

a) $2x + yi = -14 - 3i$

$2x = -14$

$x = -7$

$yi = -3i$

$y = -3$

b) $5x + 1 + (3 + 2y)i = 2x - 2 + (y - 6)i$

$5x + 1 + 3i + 2yi = 2x - 2 + yi - 6i$

$5x + 1 = 2x - 2$

$3x + 1 = -2$

$3x = -3$

$x = -1$

$3i + 2yi = yi - 6i$

$-yi \quad -yi$

$3i + yi = -6i$

$yi = -9i$

$y = -9$

c) $20 - 12i = 5y + 4xi$

$20 = 5y$

$4 = y$

$\frac{-12i}{4i} = \frac{4xi}{4i}$

$-3 = x$

Ex. 5 - Multiply.

a) $(2 + i)(3 - i)$

$6 - 2i + 3i - i^2$

$6 + i - i^2$

$6 + i + (+1)$

$7 + i$

b) $(5 - 2i)(4 - i)$

$20 - 5i - 8i + 2i^2$

$20 - 13i + 2(-1)$

$18 - 13i$

c) $(4 - 2i)(1 - 2i)$

d) $(2 + 3i)(2 - 3i)$

$4 - \cancel{6i} + \cancel{6i} - 9i^2$

$4 - 9(-1)$

$4 + 9 = 13$

Ex. 6 - Divide.

a) $\dfrac{5i}{3 + 2i}\cdot\dfrac{(3 - 2i)}{(3 - 2i)}$

$\dfrac{15i - 10i^2}{9 + 4} = \dfrac{15i + 10}{13}$

b) $\dfrac{5}{3 + i}\cdot\dfrac{(3 - i)}{(3 - i)}$

$\dfrac{15 - 5i}{9 + 1} = \dfrac{15 - 5i}{10}$

$\dfrac{3 - i}{2}$

c) $\dfrac{5 + i}{2i}\cdot\dfrac{i}{i} = \dfrac{5i + i^2}{-2}$

$\dfrac{5i - 1}{-2}$

d) $\dfrac{6 - 5i}{3i}\cdot\dfrac{i}{i} = \dfrac{6i - 5i^2}{-3}$

$\dfrac{6i + 5}{-3}$

$-2i - \dfrac{5}{3}$

e) $\dfrac{(2 + i)^2}{4 - i}$

$(2 + i)(2 + i)$

$4 + 4i + i^2$

$4 + 4i + (-1) = 3 + 4i$

$\dfrac{3 + 4i}{4 - i}\cdot\dfrac{4 + i}{4 + i} = \dfrac{12 + 3i + 16i + 4i^2}{16 - i^2} = \dfrac{12 + 19i - 4}{17}$

$\dfrac{8 + 19i}{17}$

Homework:

p. 280 (#18-58 Evens, 66, 67, 71, 72, 74)